Generalizing CDCL with Graph Backtracking

SAT Presentation

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Acknowledgements

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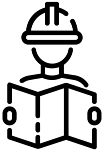
Introduction



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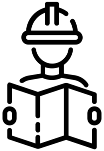

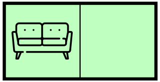

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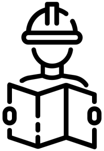

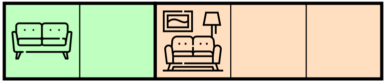

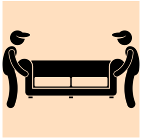

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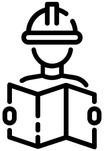

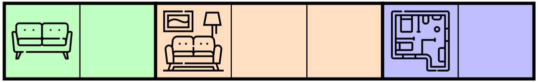

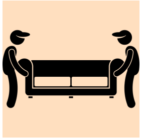

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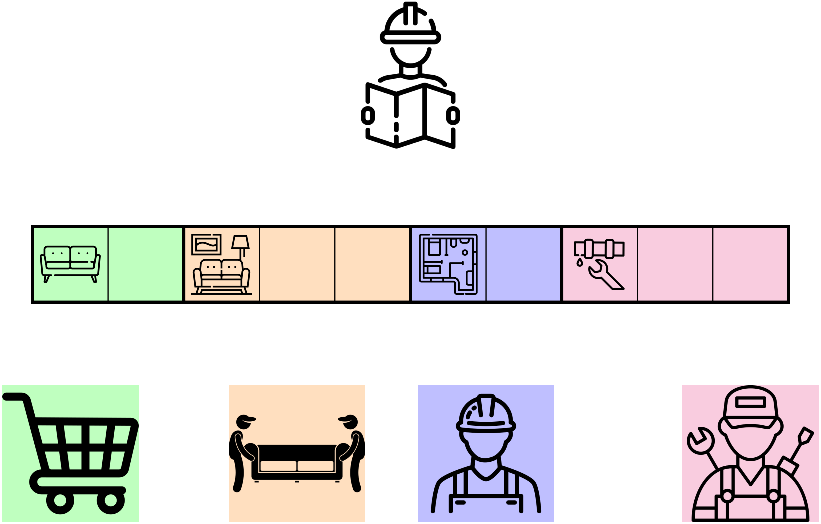

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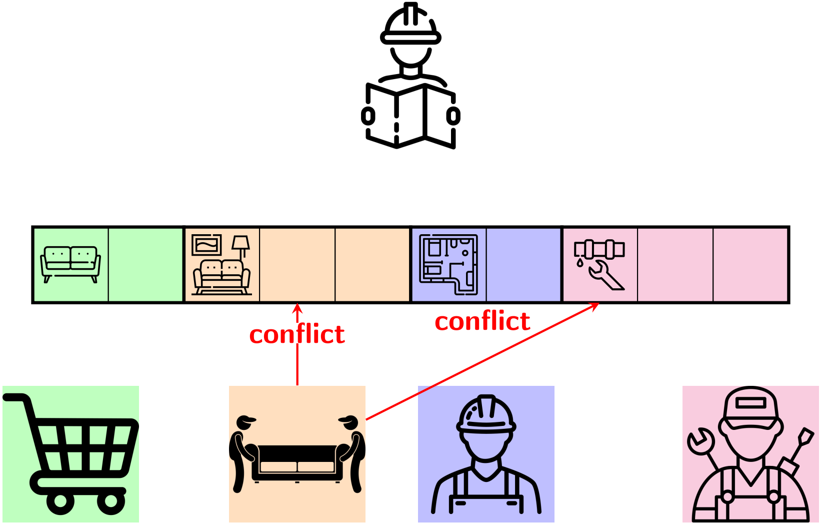

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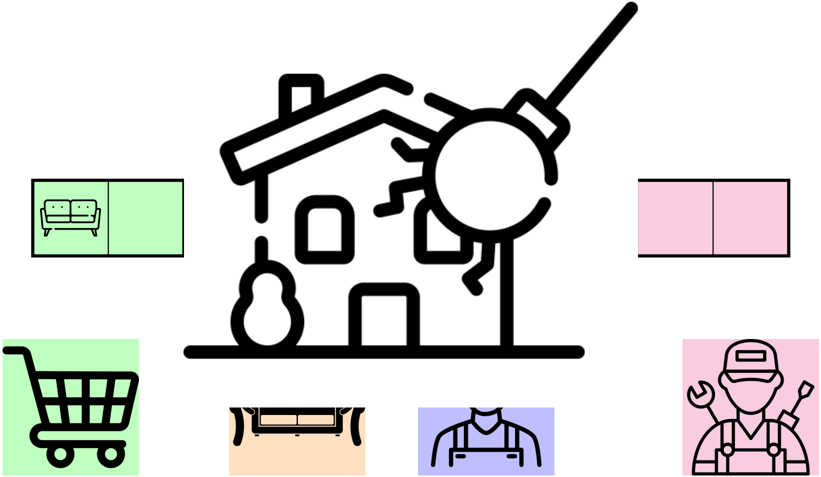

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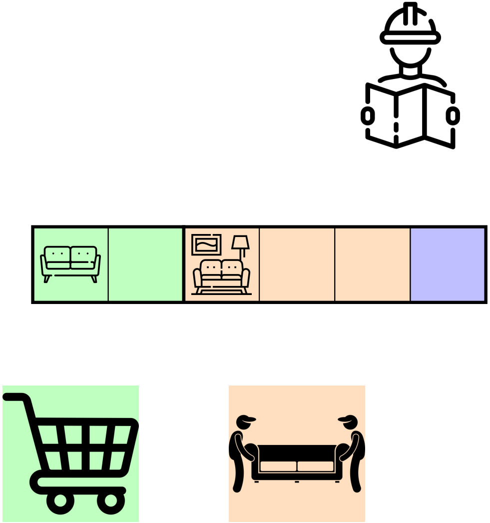

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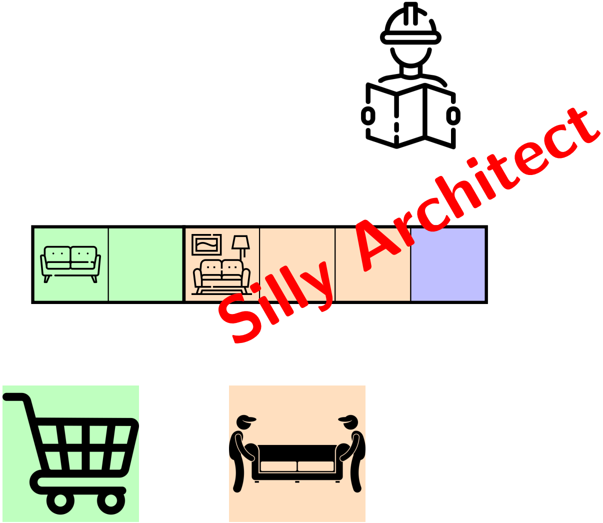
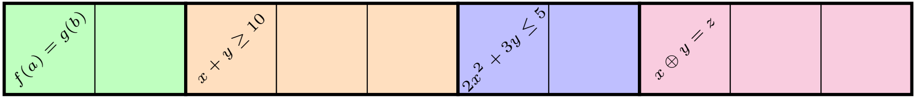


SMT Architecture

SAT Solver



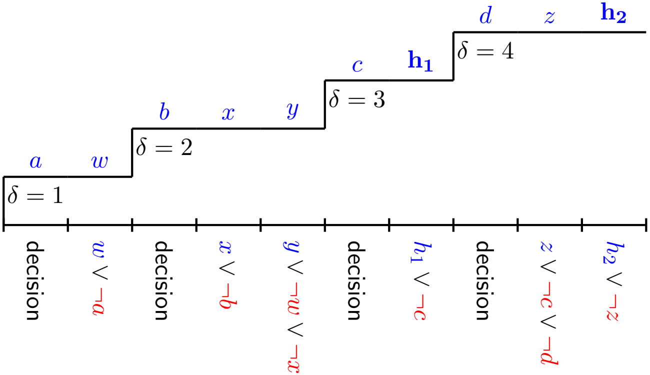
UF
(Uninterpreted Functions)

LRA
(Non-linear Real Arithmetic)

NRA
(Linear Real Arithmetic)

BV
(Bit Vectors)

Non-Chronological Backtracking – Trail



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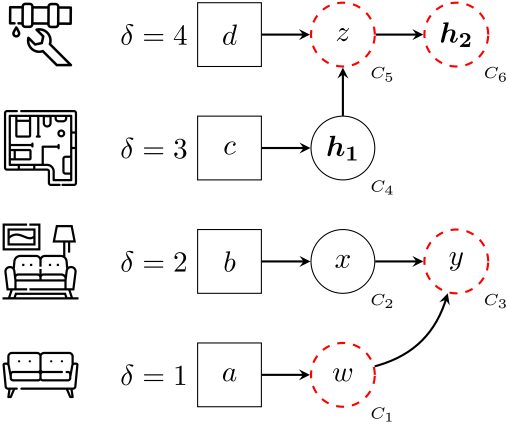
Non-Chronological Backtracking – Implication Graph



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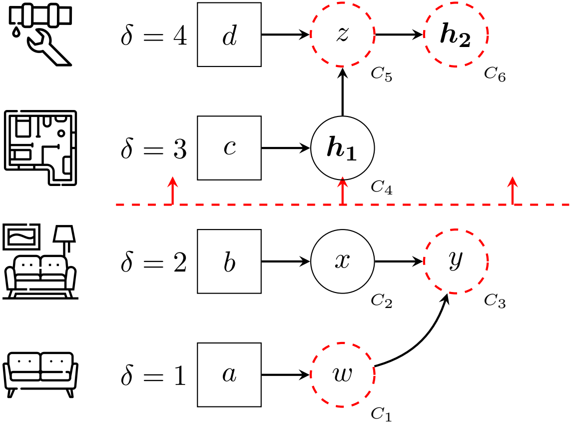

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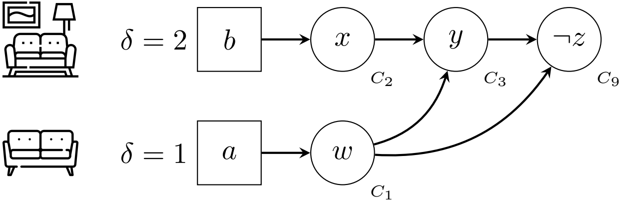

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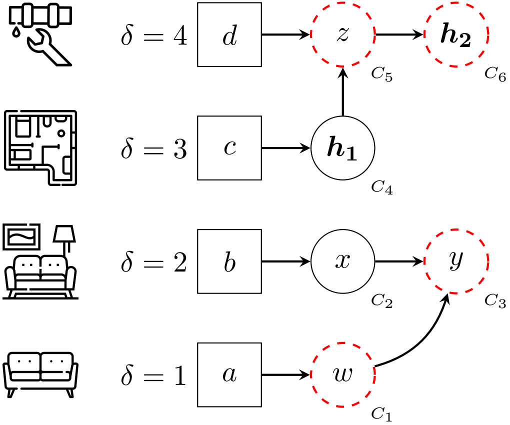
Chronological Backtracking [NR18, MB19, Nad22, CFK24]



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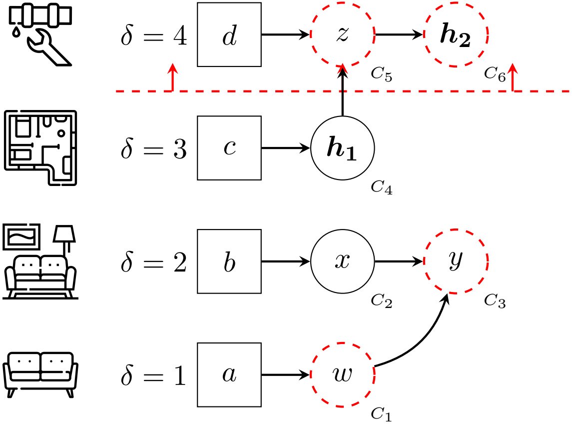

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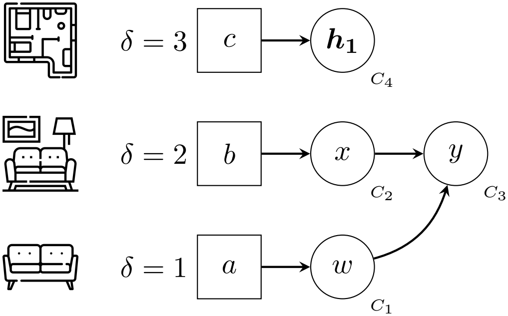

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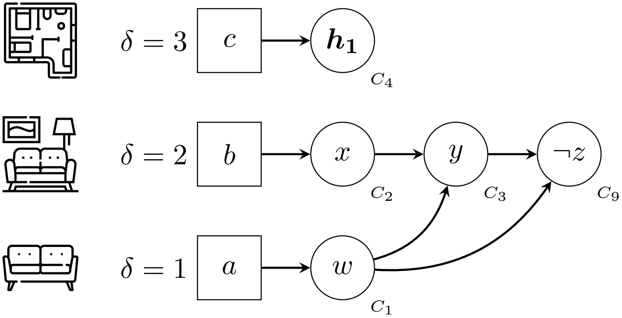

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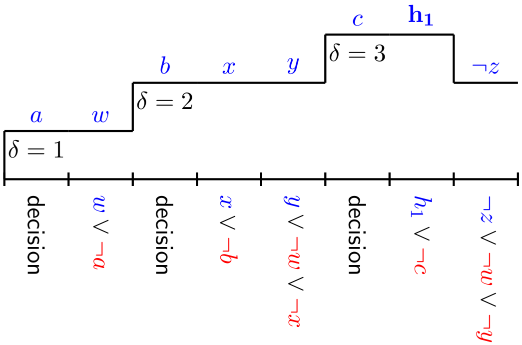

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To be fast

- NCB maintains nice **invariants**
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To be general

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- The cost of literals is unknown to the SAT solver

To be smart

- Aggressive backjumping can undo a lot of useless work
- Maybe learn fewer clauses

A More Surgical Architect

Extended Interface

We allow the user to provide a **cost** for each literal.

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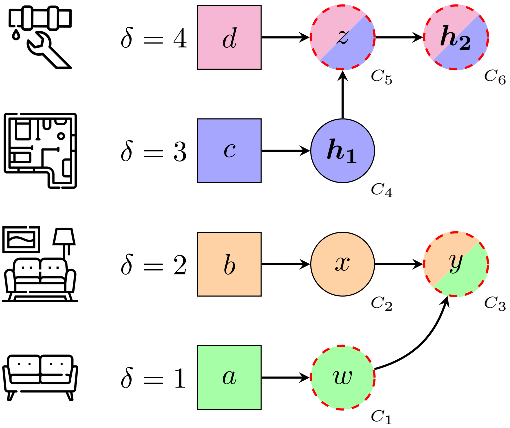
New Backtracking Scheme

Graph Backtracking (GB) searches the cheapest set of literals to backtrack to resolve a conflict. We use the implication graph to find such a set.

Graph Backtracking (GB)



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Graph Backtracking (GB)

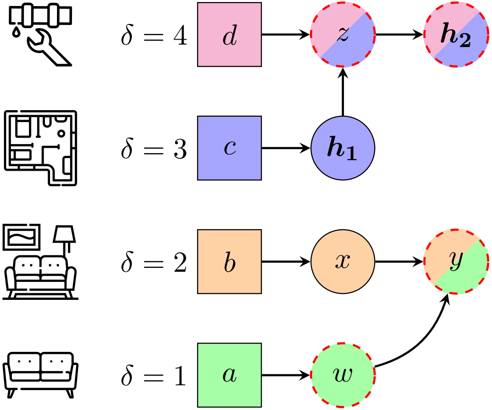
Backtrack options

option 1:

option 2:

option 3:

option 4:



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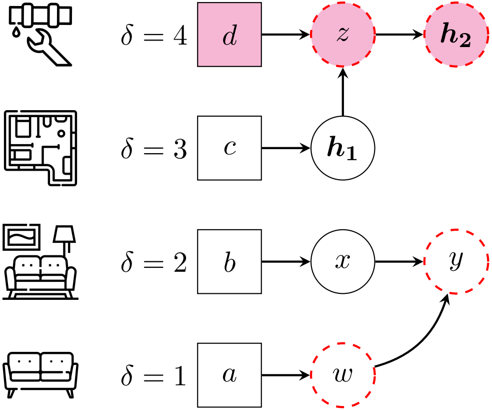

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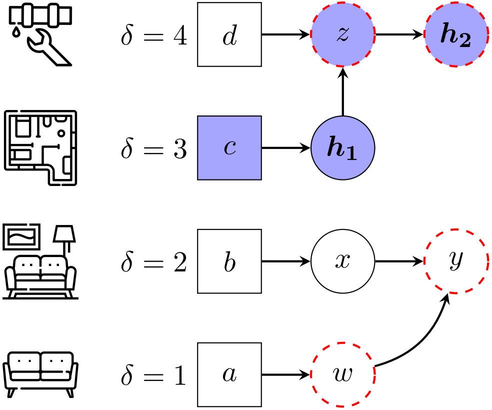

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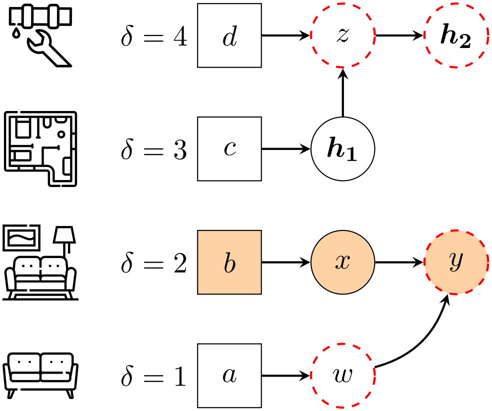

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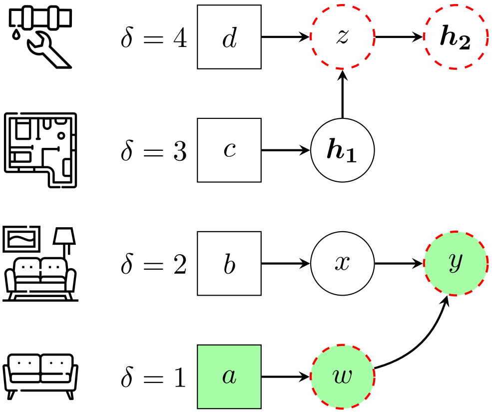

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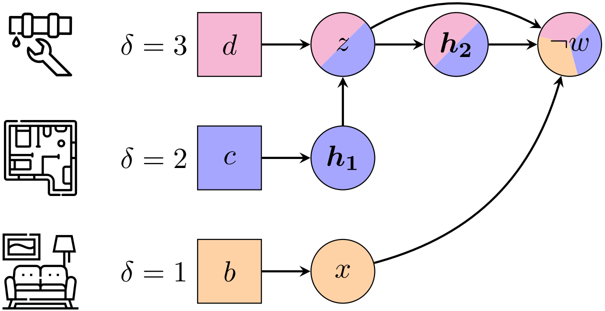
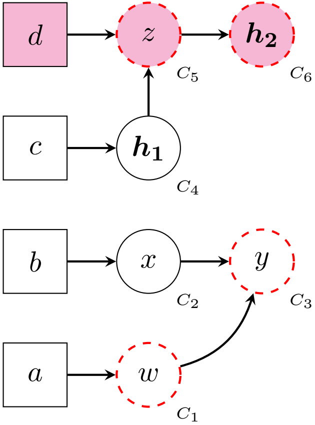


First Unique Implication Point with GB



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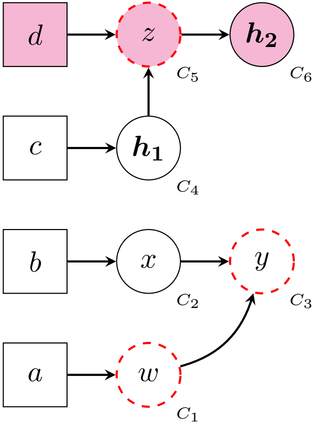

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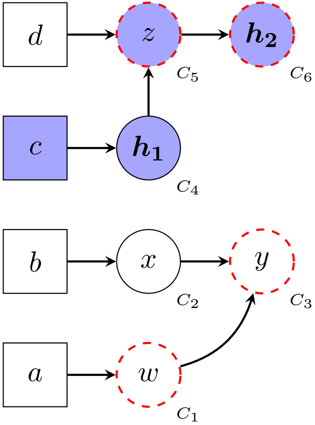

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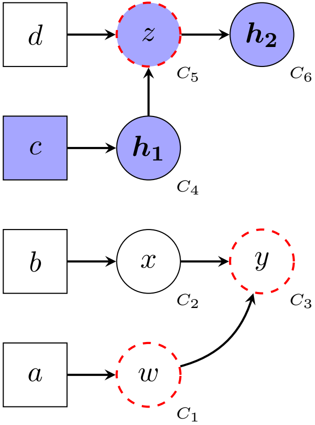

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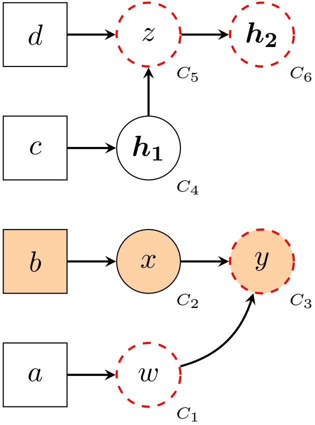

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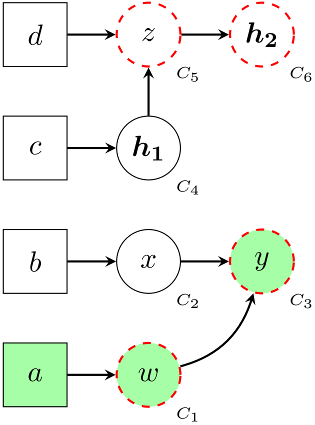

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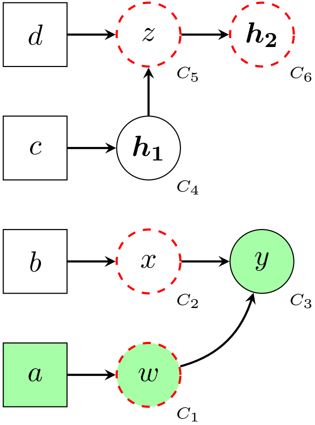

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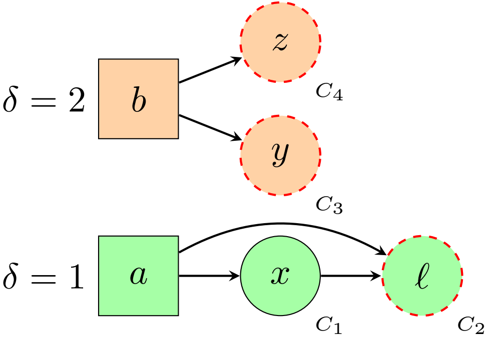

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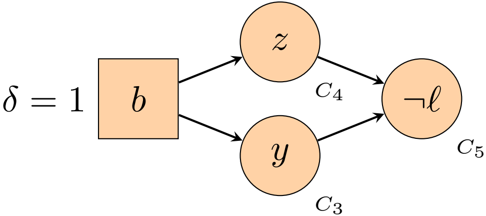

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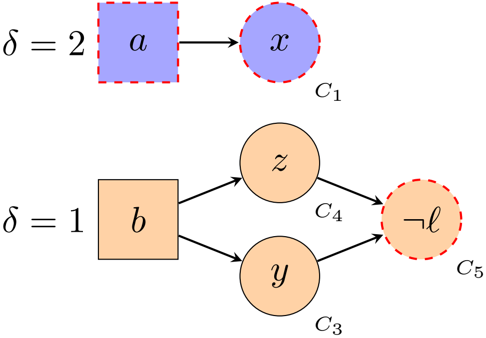

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C_1 &= x \lor \neg a \\
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C_4 &= y \lor \neg b \\
C_5 &= \neg\ell \lor \neg y \lor \neg z
\end{aligned}
$$

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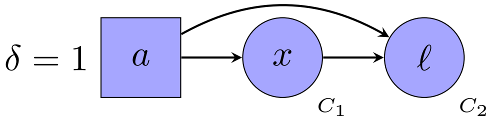

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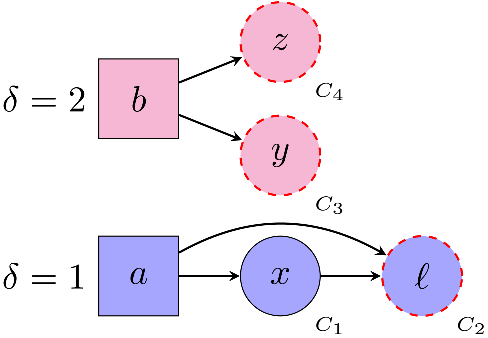

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Challenge I: Safeguards

Why this happens

- Arbitrary cost function
- No guarantee to learn a new clause after backtracking (like in CB [MB19])

Challenge I: Safeguards

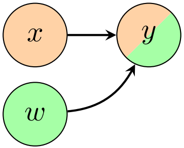
Why this happens

- Arbitrary cost function
- No guarantee to learn a new clause after backtracking (like in CB [MB19])

Solution

- If possible to learn a clause: choose the chunk that allows to learn the clause with minimum total cost
- Otherwise, select latest decision (like in CB)

Challenge II: Watched Literals



Let us watch y and $\neg x$ in C_3.

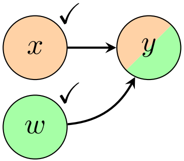
$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

Invariant (Watched Literals)

For each clause $C \in F$ watched by c_1, c_2, we have

$$\textit{propagated}(\neg c_1) \Rightarrow \textit{satisfied}(c_2)$$

Challenge II: Watched Literals



$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

Let us watch y and $\neg x$ in C_3.

$$\text{propagated}(\neg y) \Rightarrow \text{satisfied}(\neg x)$$
$$\text{propagated}(x) \Rightarrow \text{satisfied}(y)$$

is satisfied.

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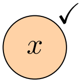

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$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

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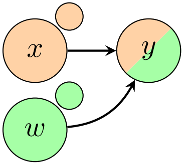
Challenge II: Watched Literals (Solution)



We define cross-colors(ℓ) as the set of colors that trigger the repropagation of ℓ.

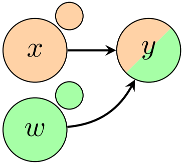
$$C_3 = \underline{y} \vee \underline{\neg x} \vee \neg w$$

Invariant (Watched Literals)

For each clause $C \in F$ watched by c_1, c_2, we have

$$\textit{propagated}(\neg c_1) \Rightarrow \left[\textit{satisfied}(c_2) \wedge \textit{colors}(c_2) \subseteq \textit{cross-colors}(c_1)\right]$$

Challenge II: Watched Literals (Solution)



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Let us watch y and $\neg x$ in C_3.

$$\text{propagated}(\neg y) \Rightarrow \left[\text{satisfied}(\neg x) \wedge \quad \subseteq \quad \right]$$

$$\text{propagated}(x) \Rightarrow \left[\text{satisfied}(y) \wedge \quad \subseteq \quad \right]$$

is satisfied.

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For each clause $C \in F$ watched by c_1, c_2, we have

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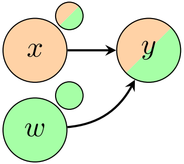

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Let us watch y and $\neg x$ in C_3.

$$\text{propagated}(\neg y) \Rightarrow \left[\text{satisfied}(\neg x) \wedge \subseteq \right]$$

$$\text{propagated}(x) \Rightarrow \left[\text{satisfied}(y) \wedge \subseteq \right]$$

is satisfied.

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For each clause $C \in F$ watched by c_1, c_2, we have

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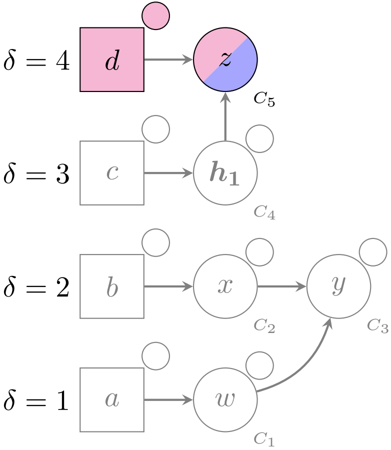
Challenge II: Cross-Coloring Example



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$C_5 = \underline{z} \vee \underline{\neg d} \vee \neg h_1$

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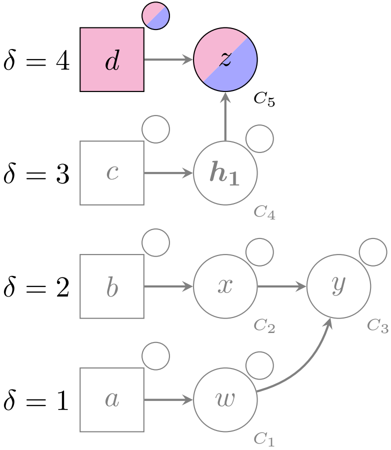

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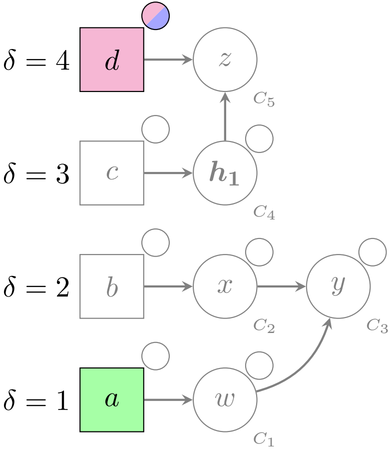

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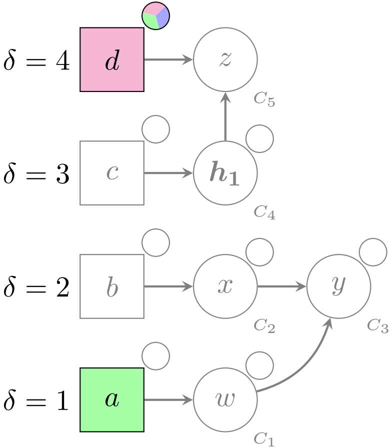

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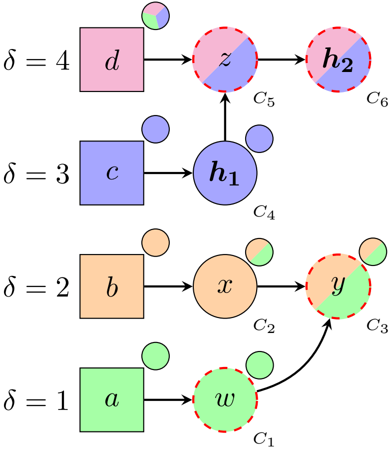

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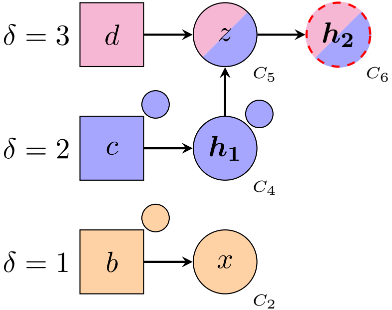

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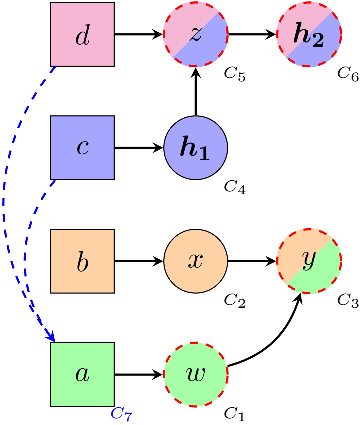
\end{aligned}
$$

Refinement: Color Merging



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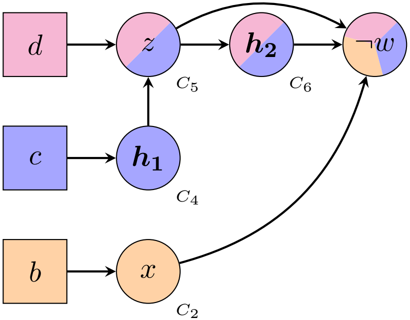

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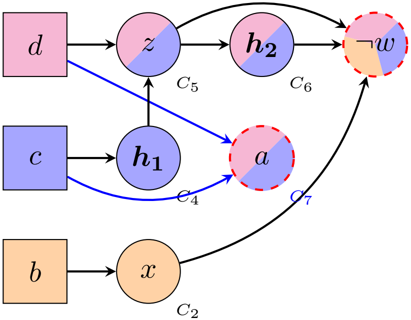

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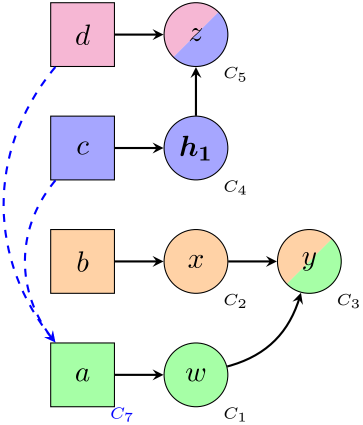
Refinement: Eager Color Merging (ECM)



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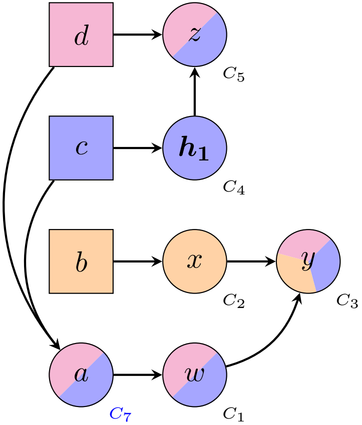

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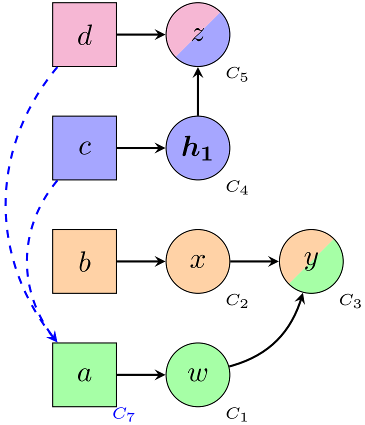
Refinement: Lazy Color Merging (LCM)

Backtrack options



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Refinement: Lazy Color Merging (LCM)

Backtrack options

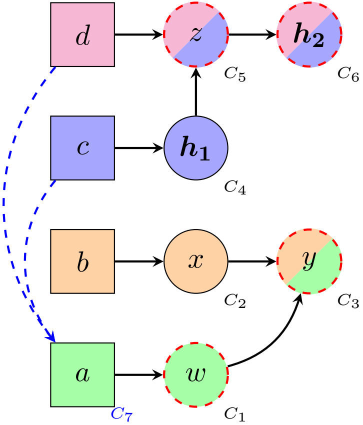
option 1:

option 2:

option 3:

option 4:

option 5:



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Refinement: Lazy Color Merging (LCM)

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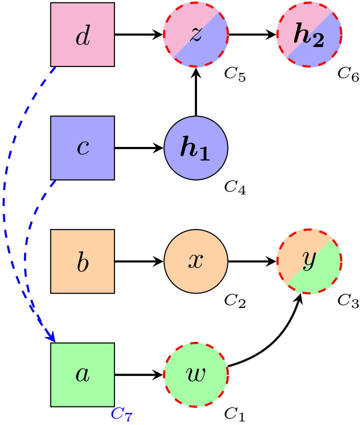

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\end{aligned}
$$

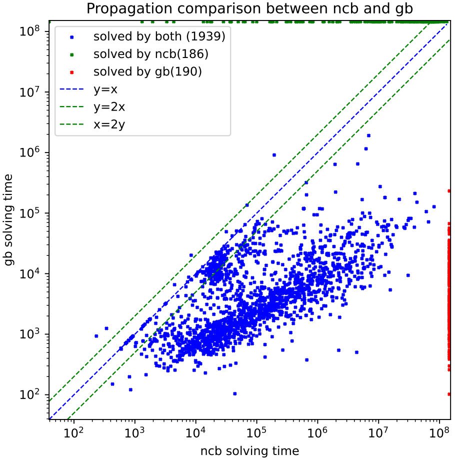
Preliminary Empirical Results

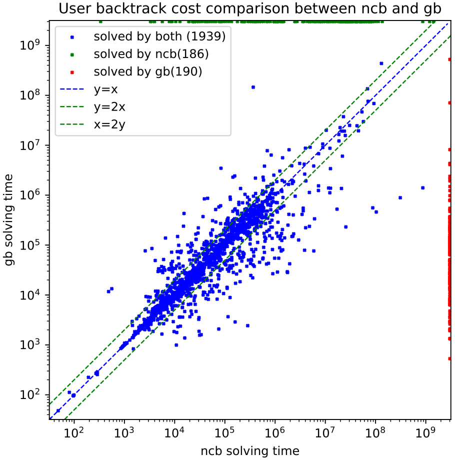
Option	# Timeout	PAR2	Time (s)		Propagations $\times 10^6$	
NCB	16	380.7	106 $\pm$ 428	-	23.7 $\pm$ 46.4	-
CB	11	333.1	103 $\pm$ 414	97.33%	22.9 $\pm$ 42.8	96.30%
GB	17	378.1	103 $\pm$ 402	96.91%	14.2 $\pm$ 25.3	60.05%
GB+ECM	11	299.7	72.4 $\pm$ 310	67.89%	12.6 $\pm$ 23.8	52.96%
GB+LCM	14	376.8	124 $\pm$ 485	117.1%	15.8 $\pm$ 29.9	66.39%

Table: Empirical results on 1000 satisfiable random 3-coloring problems with 650 nodes and 1469 edges

Results in Vampire [BBC+25] with AVATAR [Vor14]

We ran Vampire's AVATAR with NapSAT on 10617 TPTP problems requiring AVATAR.





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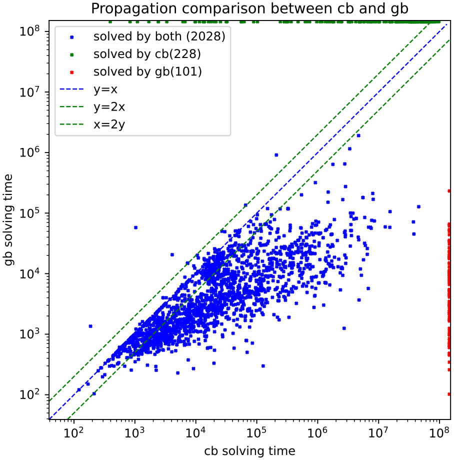

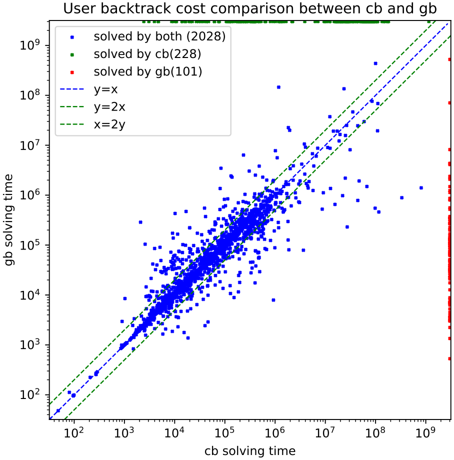

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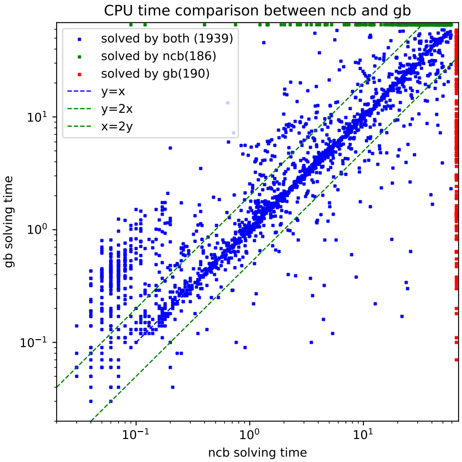

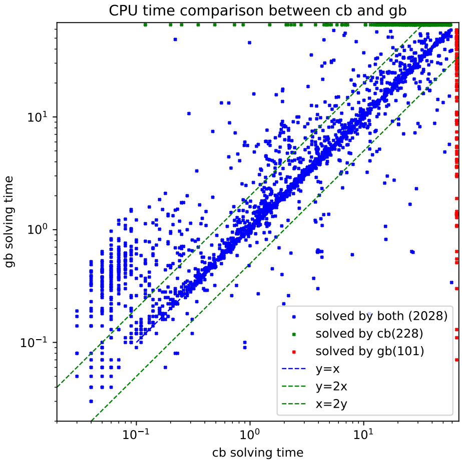

Conclusion

Summary

- Graph Backtracking (GB), a new backtracking scheme for SAT solvers
- GB extends the SAT API with cost functions
- GB is implemented in NapSAT and integrated in Vampire's AVATAR
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Future Work

- Tune Vampire AVATAR's cost heuristic to take advantage of GB.
- Improve scalability of color choice with the number of decisions.
- Relax SMT stack discipline into multi stack.
- Implement GB in cvc5 [BBB+22].

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Thank you for your attention!

References I

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